COMPLETELY CONSERVATIVE LOCALLY BAROTROPIC DIFFERENCE SCHEMES FOR GAS DYNAMICS AND MAGNETOHYDRODYNAMICS

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A new class of completely conservative difference schemes is discussed; the existence and uniqueness of a solution is guaranteed for schemes of the given class.

Implicit difference schemes, which have a large reserve of stability [1], are useful for the numerical modeling of magnetohydrodynamic flows in many cases. Problems arise, however, in connection with the algorithmic solvability of the corresponding systems of nonlinear algebraic equations with a large number of dimensions [1-3].

We have previously [4-6] investigated special classes of implicit difference schemes of gas dynamics and magnetohydrodynamics (MHD), the solvability of which is equivalent to the minimization of certain strongly convex functionals called dynamic potentials. Such schemes are referred to as locally barotropic. It has been shown that locally barotropic difference schemes have the property of complete conservatism [1] in the case of planar symmetry, i.e., difference analogs of the equations for the balance of different forms of energy are valid, along with the fundamental conservation laws, for these schemes.

Numerous test and practical computations [1, 2, 7] show that completely conservative difference schemes have a definite advantage over other schemes of the same order of approximation, a fact that is most conspicuously evident on coarse space–time grids.

In the present article we propose completely conservative locally barotropic MHD difference schemes in Lagrangian variables for the cases of planar and axial symmetry, along with completely conservative locally barotropic gasdynamic difference schemes in application to computations of spherically symmetrical flows.

1. DIFFERENTIAL EQUATIONS

The evolution of a perfectly conducting MHD fluid in the one-dimensional adiabatic approximation for the case of planar and axial symmetry in the presence of a two-component magnetic field $\mathbf{H} = (0, H_y, H_z)$ is described by the equations [1]

$$\tilde{\rho}\frac{\partial u}{\partial t} + x^{l-1}\frac{\partial}{\partial \alpha}\left(p + \frac{H_y^2 + H_z^2}{8\pi}\right) = 0, \tag{1}$$

$$\tilde{\rho}\frac{\partial \varepsilon}{\partial t} + p\frac{\partial}{\partial \alpha}(x^{l-1}u) = 0, \tag{2}$$

$$\frac{\partial x}{\partial t} = u, \tag{3}$$

$$\rho\Delta = \tilde{\rho}(\alpha), \tag{4}$$

$$H_y = \frac{1}{J}\frac{\partial A}{\partial \alpha},\quad H_z = \frac{1}{\Delta}\frac{\partial B}{\partial \alpha},\quad J = \frac{\partial x}{\partial \alpha},\quad \Delta = x^{l-1}J, \tag{5}$$

$$p = \mathcal{P}(\rho, \varepsilon). \tag{6}$$

Equations (1)-(6) describe the problem with planar symmetry for $\ell = 1$, and with axial symmetry for $\ell = 2$. For $\ell = 2$ the coordinate x is radial, and H_y denotes the azimuthal component of the field.

We seek a solution of Eqs. (1)-(6) in the domain $\Omega = \{0 \leq \alpha \leq 1,\ t \geq 0\}$, with the following boundary conditions specified at the boundary:

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$$\left(p + \frac{H_y^2 + H_z^2}{8\pi}\right)\Big|_{\alpha=0} = p_0^*(x(0, t), t),$$
$$\left(p + \frac{H_y^2 + H_z^2}{8\pi}\right)\Big|_{\alpha=1} = p_1^*(x(1, t), t). \tag{7}$$

We write the initial conditions in the form

$$x(\alpha, 0) = \varphi_1(\alpha),\ u(\alpha, 0) = \varphi_2(\alpha),$$
$$\rho(\alpha, 0) = \varphi_3(\alpha),\ \varepsilon(\alpha, 0) = \varphi_4(\alpha), \tag{8}$$
$$A(\alpha, 0) = \varphi_5(\alpha),\ B(\alpha, 0) = \varphi_6(\alpha).$$

Here, according to Eqs. (4) and (5), the following compatibility conditions must be satisfied:

$$\varphi_3(\alpha)\varphi_1^{l-1}(\alpha)\frac{\partial\varphi_1(\alpha)}{\partial\alpha} = \tilde{\rho}(\alpha),$$

$$H_y(\alpha, 0) = \frac{\partial\varphi_5(\alpha)}{\partial\alpha} \Big/ \frac{\partial\varphi_1(\alpha)}{\partial\alpha},\ H_z(\alpha, 0) = \frac{\partial\varphi_6(\alpha)}{\partial\alpha} \Big/ \left(\varphi_1^{l-1}(\alpha)\frac{\partial\varphi_1(\alpha)}{\partial\alpha}\right).$$

2. FAMILY OF LOCALLY BAROTROPIC DIFFERENCE SCHEMES

In the domain $\bar{\Omega}$ we introduce a uniform grid $\bar{\omega}_{h\tau} = \bar{\omega}_h \times \omega_\tau$. We associate with the nodes $\bar{\omega}_h$ of the grid (i.e., with the set $\bar{\omega}$) the particle coordinate and velocity functions of a continuum, as well as the magnitudes of the magnetic fluxes: u, x, A, B $\in \mathcal{H}_{\bar{\omega}}$, and we refer all thermodynamic variables and components of the magnetic field vector to the set ω: $\tilde{\rho}$, ρ, p, ε, H_y, $H_z \in \mathcal{H}_\omega$.

In accordance with [6], we define the operators $\langle\langle\cdot\rangle\rangle$ and $\langle\langle\cdot\rangle\rangle^*$, which project the grid functions from $\mathcal{H}_{\bar{\omega}}$ into $\mathcal{H}_\omega$ and from $\mathcal{H}_\omega$ into $\mathcal{H}_{\bar{\omega}}$ and are such that

$$\sum_{i\in\omega} g_i \langle\langle f\rangle\rangle_i = \sum_{j\in\bar{\omega}} f_j \langle\langle g\rangle\rangle_j^* \tag{9}$$

for any $f \in \mathcal{H}_{\bar{\omega}}$ and $g \in \mathcal{H}_\omega$.

We also consider the linear operators $\langle\partial f/\partial\alpha\rangle$: $\mathcal{H}_{\bar{\omega}} \to \mathcal{H}_{\omega^+}$ and $\langle\partial g/\partial\alpha\rangle^*$: $\mathcal{H}_{\omega^+} \to \mathcal{H}_{\bar{\omega}}$, which approximate the enclosed derivative at the points of the corresponding set and are such that for any functions $f \in \mathcal{H}_{\bar{\omega}}$ and $g \in \mathcal{H}_{\omega^+}$

$$\tilde{g}_0 f_0 - g_N f_N + \sum_{i\in\omega} g_i < \frac{\partial f}{\partial\alpha} >_i h_\alpha = -\sum_{j\in\bar{\omega}} f_j < \frac{\partial g}{\partial\alpha} >_j^* h_\alpha, \tag{10}$$

where

$$\tilde{g}_0 = \sum_{i\in\omega} g_i \left[\frac{\partial}{\partial f_0} < \frac{\partial f}{\partial\alpha} >_i h_\alpha\right]. \tag{11}$$

We assume that the continuum is locally barotropic [4-6], i.e., at every point $\alpha \in \Omega$ for values $t \in (t_n, t_{n+1}]$ the thermodynamic state of the medium is a function of the density ρ only, i.e., $p(\alpha, t) = P(\rho(\alpha, t), c(\alpha, t_n))$, where $c(\alpha, t_n)$ is some parameter (or several parameters) evaluated in the preceding layer $t = t_n$.

We consider the two-parameter family of locally barotropic difference schemes [6]

$$Mu_t = -\left[x^{l-1} < \frac{\partial}{\partial\alpha}\left(p + \frac{H_y^2 + H_z^2}{8\pi}\right) >\right]^{(\sigma_1)} h_\alpha + \tag{12}$$

$$+ \delta_0 \frac{\partial F_0(\hat{x}_0, t_{n+1})}{\partial \hat{x}_0} - \delta_N \frac{\partial F_1(\hat{x}_N, t_{n+1})}{\partial \hat{x}_N}, \tag{13}$$

$$m\varepsilon_t = -[pS(x^{l-1} u^{(0,5)})]^{(\sigma_1)} + Q_H, \tag{14}$$

$$x_t = u^{(\sigma_2)}, \tag{15}$$

$$\rho V = m,$$

$$H_y = \Delta A/S,\ H_z = \Delta B/V, \tag{16}$$

$$\hat{p} = \mathscr{P}(\hat{\rho},\ \hat{\varepsilon}), \tag{17}$$

$$\hat{p} = P(\hat{\rho},\ c). \tag{18}$$

The functions F_0 and F_1 introduced in Eq. (12) are defined as follows:

$$\left[x^{l-1}\left(p + \frac{H_y^2 + H_z^2}{8\pi}\right)\right]^{(\sigma_1)}\Bigg|_{i=0} = [p_0^*(x_0,\ t)\, x_0^{l-1}]^{(\sigma_1)} = \frac{\partial F_0(\hat{x}_0,\ t_{n+1})}{\partial \hat{x}_0},$$

$$\left[x^{l-1}\left(p + \frac{H_y^2 + H_z^2}{8\pi}\right)\right]^{(\sigma_1)}\Bigg|_{i=N} = [p_1^*(x_N,\ t) x_N^{l-1}]^{(\sigma_1)} = \frac{\partial F_1(\hat{x}_N,\ t_{n+1})}{\partial \hat{x}_N}.$$

The system of equations (13)-(18) represents a closed conservative system of MHD difference equations [6]. It has been shown [6] that for the given approximation (12) of the dynamical equation (1), even in our case (with $\sigma_1 = \sigma_2 = 0.5$), $Q_H = O(\tau_n^2) \neq 0$, i.e., a redistribution of energy takes place between its distinct forms: magnetic and thermal. This means that the scheme (12)-(18) is not completely conservative [1].

3. ONE-PARAMETER FAMILY OF COMPLETELY CONSERVATIVE LOCALLY BAROTROPIC DIFFERENCE SCHEMES IN THE PRESENCE OF A SINGLE-COMPONENT MAGNETIC FIELD ($H_z \neq 0$, $H_y \equiv 0$)

We show that the difference equations

$$Mu_t = -\hat{x}^{l-1} < \frac{\partial}{\partial \alpha}\left(p^{(\sigma_1)} + \frac{H_z \hat{H}_z}{8\pi}\right) >^* h_\alpha + \delta_0 \frac{\partial \tilde{F}_0(\hat{x}_0,\ t_{n+1})}{\partial \hat{x}_0} - \delta_N \frac{\partial \tilde{F}_1(\hat{x}_N,\ t_{n+1})}{\partial \hat{x}_N}, \tag{12'}$$

$$m\varepsilon_t = -p^{(\sigma_1)} V_t, \tag{13'}$$

$$x_t^l = l\hat{x}^{l-1} u^{(0,5)}, \tag{14'}$$

$$\rho V = m, \tag{15}$$

$$H_z = \Delta B/V, \tag{16}$$

$$\hat{p} = \mathscr{P}(\hat{\rho},\ \hat{\varepsilon}), \tag{17}$$

$$\hat{p} = P(\hat{\rho},\ c) \tag{18}$$

represent a completely conservative MHD scheme.

The functions $\tilde{F}_0$ and $\tilde{F}_1$ introduced in Eq. (12') are determined from the relations

$$\left[\hat{x}^{l-1}\left(p^{(\sigma_1)} + \frac{H_z \hat{H}_z}{8\pi}\right)\right]\Bigg|_{i=0} = [p_0^*(x_0,\ t)]^{(\sigma_1)}\, \hat{x}_0^{l-1} = \frac{\partial \tilde{F}_0(\hat{x}_0,\ t_{n+1})}{\partial \hat{x}_0},$$
$$\left[\hat{x}^{l-1}\left(p^{(\sigma_1)} + \frac{H_z H_z}{8\pi}\right)\right]\Bigg|_{i=N} = [p_1^*(x_N,\ t)]^{(\sigma_1)}\, \hat{x}_N^{l-1} = \frac{\partial \tilde{F}_1(\hat{x}_N,\ t_{n+1})}{\partial \hat{x}_N}. \tag{19}$$

The variation of the magnetic field energy E_H during the time τ_n is given by the expression

$$(E_H)_t = \frac{1}{8\pi}(H_z^2 V)_t.$$

Making use of the frozen-in condition (16), we obtain an equation for the balance of magnetic energy on the layer:

$$(E_H)_t = -\frac{H_z \hat{H}_z}{8\pi} V_t = -\frac{H_z \hat{H}_z}{8\pi} < \frac{\partial(\hat{x}^{l-1} u^{(0,5)})}{\partial \alpha} > h_\alpha. \tag{20}$$

Multiplying the equation of motion (12') by $u^{(0.5)}$, we obtain an equation for the variation of the kinetic energy of a "fluid particle" $E_K = \tilde{\rho}u^2/2 = \rho V u^2/2$ during the time τ_n:

$$(E_K)_t = -\left[\hat{x}^{l-1} < \frac{\partial}{\partial \alpha}\left(p^{(\sigma_1)} + \frac{H_z \hat{H}_z}{8\pi}\right) >^* h_\alpha - \delta_0 \frac{\partial \tilde{F}_0(\hat{x}_0,\ t_{n+1})}{\partial \hat{x}_0} + \delta_N \frac{\partial \tilde{F}_1(\hat{x}_N,\ t_{n+1})}{\partial \hat{x}_N}\right] u^{(0,5)}. \tag{21}$$

We multiply the balance equations for the magnetic (20) and kinetic energy (21):

$$(E_\text{K} + E_H)_t = -\frac{H_z\hat{H}_z}{8\pi} < \frac{\partial \hat{x}^{l-1} u^{(0,5)}}{\partial \alpha} > h_\alpha - \hat{x}^{l-1} \times \tag{22}$$

$$\times u^{(0,5)} < \frac{\partial}{\partial \alpha} (p^{(\sigma_1)} + \frac{H_z\hat{H}_z}{8\pi}) >^* h_\alpha + \left[\delta_0 \frac{\partial \tilde{F}_0(\hat{x}_0, t_{n+1})}{\partial \hat{x}_0} - \delta_N \frac{\partial \tilde{F}_1(\hat{x}_N, t_{n+1})}{\partial \hat{x}_N} \right] u^{(0,5)}.$$

We sum the resulting equation over the grid $\bar{\omega}_h$ and take into account the definition of the functions $\tilde{F}_0$ and $\tilde{F}_1$ (19). We obtain the law of conservation of the total energy

$$\sum_{i\in\omega} (E_\text{K} + E_H)_{t,i} = \sum_{i\in\omega} p_i^{(\sigma_1)} < \frac{\partial \hat{x}^{l-1}}{\partial \alpha} u^{(0,5)} >_i h_\alpha.$$

We thus have shown that not only the finite-difference laws of conservation of mass (15), momentum (12'), and total energy (22) as in the case of classical conservative scheme, but also a number of additional grid relations hold for the family of difference schemes (12')-(14'), (15)-(18), viz.: the laws of conservation of specific internal energy (13'), magnetic energy (20), and kinetic energy (21) and the law governing the volume variation, which has the form (15) for the given scheme, i.e., the scheme (12')-(14'), (15)-(18) is completely conservative.

4. DISCRETE DYNAMIC POTENTIALS AND THEIR PROPERTIES

We analyze the assumptions regarding the parameters τ_n and σ_1 in order for a solution of the system of equations (12')-(14'), (15)-(18) to exist and be unique.

Let $\delta\hat{x} \in \mathcal{H}_{\bar{\omega}}$ be an infinitesimal variation of the grid function $\hat{x} \in \mathcal{H}_{\bar{\omega}}$. We multiply (12') by $\delta\hat{x}$ and sum over the grid $\bar{\omega}$:

$$\delta\Phi_{n+1} = \sum_{i\in\omega} \left[m_i \frac{\langle\langle \hat{u}\,\delta\hat{x} - u\delta\hat{x} \rangle\rangle_i}{\tau_n} - \left(p^{(\sigma_1)} + \frac{H_z\hat{H}_z}{8\pi}\right)_i \delta\hat{V}_i \right] - \tag{23}$$

$$- p_0^{*(\sigma_1)} \hat{x}_0^{l-1} \delta\hat{x}_0 + p_1^{*(\sigma_1)} \hat{x}_N^{l-1} \delta\hat{x}_N.$$

Substituting

$$\hat{u} = 2\frac{\hat{x}^l - x^l}{l\hat{x}^{l-1}\tau_n} - u$$

and

$$p^{(\sigma_1)} = \sigma_1 \hat{\rho}^2 \frac{\partial \mathcal{F}(\alpha, \hat{\rho})}{\partial \hat{\rho}} + (1-\sigma_1)p, \quad \mathcal{F}(\alpha, \rho) =$$

$$= \int_0^\rho \frac{P(\xi, c(\alpha, t_n))}{\xi^2} d\xi > 0$$

in Eq. (23), we obtain

$$\Phi_{n+1}(\hat{x}) = \Phi_{n+1}^\text{K}(\hat{x}) + \Phi_{n+1}^p(\hat{x}) + \Phi_{n+1}^H(\hat{x}) - \tilde{F}_0(\hat{x}_0, t_{n+1}) + \tilde{F}_1(\hat{x}_N, t_{n+1}), \tag{24}$$

where

$$\Phi_{n+1}^\text{K}(\hat{x}) = \sum_{i\in\omega} \frac{2m_i}{\tau_n} \left\{ \frac{\langle\langle \hat{x}^2 \rangle\rangle_i}{2l\tau_n} + \frac{1}{l\tau_n} [(l-2) \langle\langle x^l \hat{x}^{-(l-2)} \rangle\rangle_i + \right.$$

$$\left. + (l-1) \langle\langle x^l \ln \hat{x} \rangle\rangle_i] - \langle\langle u\hat{x} \rangle\rangle_i \right\};$$

$$\Phi_{n+1}^p(\hat{x}) = \sum_{i\in\omega} [\sigma_1 m_i \mathcal{F}_i(\alpha, \hat{\rho}) - (1-\sigma_1) p_i \hat{V}_i];$$

$$\Phi_{n+1}^H(\hat{x}) = -\sum_{i\in\omega} \frac{1}{8\pi V_i} (\Delta B)_i^2 \ln \left(\frac{\hat{V}_i}{h_\alpha} \right).$$

The quantities $\Phi_{n+1}(\hat{x})$ are called discrete dynamic potentials, and $\Phi^\text{K}_{n+1}(\hat{x})$, $\Phi^p_{n+1}(\hat{x})$, and $\Phi^H_{n+1}(\hat{x})$ are called their kinetic, thermodynamic, and magnetic components [1-3]. Consequently, the dynamic equation (12') determines a stationary point of the functional $\Phi_{n+1}(\hat{x})$ with respect to the variables $\hat{x}_i$, $i \in \omega$. This means that Eq. (12') can be written

$$\frac{\partial \Phi_{n+1}(x_i)}{\partial \hat{x}_j} = 0, \quad i, j \in \bar{\omega}. \tag{12''}$$

We analyze the functional $\Phi_{n+1}(\hat{x})$ inside the convex cone $\mathscr{K} = \mathscr{K}(t_{n+1}) = \{\hat{x} \in \mathscr{H}_{\bar{\omega}} : V(\hat{x}) > 0\}$. It is obviously continuous on the set $\mathscr{K}$ and tends to $+\infty$ at its boundaries.

The second differential of $\Phi_{n+1}(\hat{x})$ has the form

$$d^2\Phi_{n+1}(\hat{x}) = d^2\Phi^{\kappa}_{n+1}(\hat{x}) + d^2\Phi^{p}_{n+1}(\hat{x}) + d^2\Phi^{H}_{n+1}(\hat{x}) - \\ - \frac{\partial^2 \tilde{F}_0}{\partial \hat{x}_0^2} d\hat{x}_0^2 + \frac{\partial^2 \tilde{F}_1}{\partial \hat{x}_N^2} d\hat{x}_N^2,$$

where

$$d^2\Phi^{\kappa}_{n+1}(\hat{x}) = \sum_{i\in\omega} \frac{2m_i}{\tau_n}\left[\frac{\langle\langle d\hat{x}\rangle\rangle_i^2}{l\tau_n} - \frac{(l-1)}{l\tau_n}\langle\langle \frac{x^l d\hat{x}^2}{\hat{x}^2}\rangle\rangle_i\right];$$

$$d^2\Phi^{p}_{n+1}(\hat{x}) = \sum_{i\in\omega}\left[\sigma_1 m_i \frac{\partial^2 \mathscr{F}_i}{\partial \hat{x}_i^2} d\hat{x}_i^2 + (1-\sigma_1)(l-1)p_i d\hat{V}_i^2\right]; \tag{25}$$

$$d^2\Phi^{H}_{n+1}(\hat{x}) = \sum_{i\in\omega} \frac{(\Delta B)_i^2}{8\pi V_i} \frac{h_\alpha}{\hat{V}_i^2} d\hat{V}_i^2.$$

It is evident from expressions (25) that if the conditions

$$\hat{x}^2 \geqslant (l-1)x^l, \\ \frac{\partial^2 \mathscr{F}}{\partial \hat{x}^2} \geqslant 0, \quad \frac{\partial^2 \tilde{F}_0}{\partial \hat{x}_0^2} \leqslant 0, \quad \frac{\partial^2 \tilde{F}_1}{\partial \hat{x}_N^2} \geqslant 0 \tag{26}$$

are satisfied and if $\sigma_1 \geqq 0$ for $\ell = 1$, $\sigma_1 \geqq 1$ for $\ell = 2$, the inequality $d^2\Phi_{n+1}(\hat{x}) > 0$ will hold, i.e., the functional $\Phi_{n+1}(\hat{x})$ is strongly convex inside the convex cone $\mathscr{K}$, and so the dynamic equation (12') determines the unique point at which the functional has a global minimum [8].

We note that in the absence of a magnetic field ($H_z \equiv H_y \equiv 0$) it is also possible to construct completely conservative locally barotropic gasdynamic difference schemes for $\ell = 3$. In this case the dynamic potential $\Phi_{n+1}(\hat{x})$ is given by expression (24) for $\Phi^H_{n+1}(\hat{x}) \equiv 0$. It is obvious that $d^2\Phi_{n+1}(\hat{x}) > 0$ under conditions (26) and $\sigma_1 \geqq 0$ for $\ell = 1$, $\sigma_1 \geqq 1$ for $\ell = 2$, 3.

Thus, when the kinetic equation (3) is approximated by expression (14'), it is possible to construct completely conservative locally barotropic gasdynamic and MHD difference schemes. We note that a difference approximation of Eq. (3) in the form $x_t = u^{(\sigma_2)}$ is discussed in [3-6].

It is important to note that a combination of the paraboloid method and the steepest-descent method can be used for the numerical determination of the minima of the dynamic potentials [9].

NOTATION

t, time; x, Eulerian variable; α, Lagrangian variable; ρ, density of medium; $\tilde{\rho} = \tilde{\rho}(\alpha) \geqq \tilde{\rho}_0 > 0$, Lagrangian density of medium; u, velocity; p, pressure; ε, internal energy: H_y, H_z, magnetic field components; $A = A(\alpha)$, $B = B(\alpha)$, magnitudes of magnetic fluxes formed by respective components H_y, H_z; $J = \partial x/\partial\alpha$, Jacobian of transformation from Eulerian to Lagrangian variables; ℓ, integer-valued parameter, equal to 1 or 2; $\Omega = \{0 \leqq \alpha \leqq 1,\ t \geqq 0\}$, $\omega_{h\tau} = \omega_h \times \omega_\tau$, uniform grid in domain Ω, $\omega_h = \{\alpha_{i+1} = \alpha_i + h_\alpha,\ \alpha_{i+1/2} = (i + 1/2)h_\alpha,\ i = 0, 1, \ldots, N-1,\ \alpha_0 = 0,\ \alpha_N = 1\}$, $\omega_\tau = \{t_{n+1} = t_n + \tau_n,\ n = 0, 1, \ldots\}$; $\bar{\omega}$, sets of nodes of grid ω_h; ω, set of centers $\alpha_{i+1/2} = (i + 1/2)h_\alpha$ of meshes of grid ω_h, $\omega^+ = \omega \cup \partial_1\Omega$; $\partial_1\Omega$, right boundary of set Ω; $\mathscr{H}_{\bar{\omega}}$, $\mathscr{H}_\omega$ and $\mathscr{H}_{\omega^+}$, sets of grid functions specified on (respectively) $\bar{\omega}$, ω and ω^+; $\langle\partial\cdot/\partial\alpha\rangle$, linear operator projecting $\mathscr{H}_{\bar{\omega}}$ onto $\mathscr{H}_{\omega^+}$ and approximating the corresponding derivative $\langle\partial\cdot/\partial\alpha\rangle^*$, conjugate operator mapping $\mathscr{H}_{\omega^+}$ into $\mathscr{H}_{\bar{\omega}}$; $\langle\langle\cdot\rangle\rangle$, linear operator projecting grid functions from $\mathscr{H}_{\bar{\omega}}$ into $\mathscr{H}_\omega$; $\langle\langle\cdot\rangle\rangle^*$, linear operator projecting grid functions from $\mathscr{H}_\omega$ into $\mathscr{H}_{\bar{\omega}}$; $S_i(x) = \langle\partial x/d\alpha\rangle_i h_\alpha$; $V_i(x) = \ell^{-1}\langle\partial x^\ell/\partial\alpha\rangle_i h_\alpha$; $\tilde{\rho}_j(\alpha)h_\alpha = m_j$; $\langle\partial A/\partial\alpha\rangle_i h_\alpha = (\Delta A)_i$; $\langle\partial B/\partial\alpha\rangle_i h_\alpha = (\Delta B)_i$; $M_j = \langle\langle m\rangle\rangle_j^*$, $i \in \omega$, $j \in \bar{\omega}$; $y = y_i^n = y(x_i, t_n)$, $\hat{y} = y^{n+1}$, $y_t = (\hat{y} - y)/\tau_n$, $y^{(\sigma)} = \sigma\hat{y} + (1-\sigma)y$, where y is a grid function; $\sigma_1 > 0$, $\sigma_2 > 0$, weighting parameters; Q_H, imbalance term; $\delta_0 = \delta_{0i}$, $\delta_N = \delta_{Ni}$, $i \in \bar{\omega}$, δ_{ij}, Kronecker delta symbol; E_H, magnetic field energy; E_K, kinetic energy; $\Phi_{n+1}(\hat{x})$, discrete dynamic potential; $\delta\Phi_{n+1}(\hat{x})$, variation of functional $\Phi_{n+1}(\hat{x})$.

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